Error Analysis of the Unloaded Q-Factors of a Transmission-Type Resonator Measured by the Insertion Loss Method and the Return Loss Method

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Abstract — **Two type measurement methods of the unloaded Q-factor of a microwave resonator, the insertion loss method and the return loss method, are reexamined theoretically and compared experimentally. An error formula is derived to estimate the errors between the unloaded Q-factors measured by the two different methods. Measured results of a stripline resonator verified well the derived formula, and proved that the return loss method is more accurate and reliable than the traditional insertion loss method.**

I. Introduction

In many applications, like the measurements of dielectric materials and superconductors, an accurate measurement of the unloaded Q-factor of a microwave resonator is of paramount importance [1]-[5]. A transmission type circuit configuration and a network analyzer are usually used in the measurement. After we get the loaded Q-factor Q_L from the measured resonant frequency and the 3-dB bandwidth of the transmitted signal, we can then determine the unloaded Q-factor Q_u from Q_L by choosing two different methods. The first method requires the measurement of insertion loss only at the resonant frequency, and is known as the insertion loss method. The second method demands the measurement of the return loss at both the input and output of the resonator, and is referred later as the return loss method.

Although the insertion loss method is widely accepted and used in the Q-measurements, it is valid only when the couplings at the input and output of the resonator are equal to each other. However, in many practical measurements, this condition is very difficult to be satisfied, and the unequal couplings may make the measured results unreliable [3]. Instead of the insertion loss method, the return loss method is claimed to be more accurate and is recommended by some researchers [1][4]. However, neither theoretical error analysis was made to convince the readers, nor comparison of the Q_u values measured by the two different methods was provided to support the recommendations.

In this paper, we make an error analysis of the unloaded Q-factors of a transmission-type resonator measured by the insertion loss method and the return loss method. In Section II, after brief derivation of the insertion loss method and the return loss method for measuring Q_u, an error-formula is provided to estimate the errors of Q_u measured by using the two methods. In Section III, the unloaded Q_u of a stripline circular patch resonator is measured by both the insertion loss method and the return loss method with unequal couplings at the input and output. It is found that the errors of the measured Q_u by the two different methods agree well with the theoretical predictions by the error-formula derived in Section II. Examinations of the values of the measured Q_u reveal also that the return loss measurement method provides more reliable results than the insertion loss method.

II. Analysis

Fig. 1 shows the equivalent circuit of a transmission-type resonator. The series RLC resonator has a resonant frequency f_0 and an unloaded Q_u. It is coupled with external circuits through ideal transformers with transform ratios $1{:}n_1$ and $1{:}n_2$, respectively.

In order to obtain the insertion loss and return loss characteristics of the resonator, we derive the ABCD matrix of the circuit by multiplying the ABCD matrices of the cascaded left-hand transformer, the series RLC resonator, and the right-hand transformer in Fig. 1 in sequence [5]. We get the following expressions:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{n_2}{n_1} & \frac{R_0}{n_1 n_2}\left[1 + jQ_u\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \\ 0 & \frac{n_1}{n_2} \end{bmatrix} \qquad (1)$$

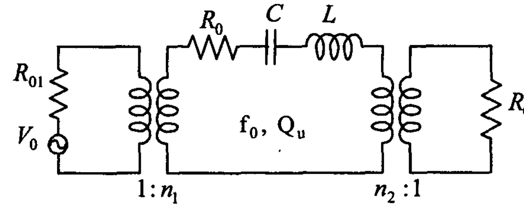

Fig. 1. Equivalent circuit of a transmission-type resonator.

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where

$$\omega_o = \frac{1}{\sqrt{LC}}, \quad Q_u = \frac{\omega_o L}{R_0}$$

By substituting the above A, B, C, and D into the conversion equations of ABCD matrix and scattering matrix [6], we get

$$S_{11}(\omega) = \frac{1-\beta_1+\beta_2+jQ_u\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}{1+\beta_1+\beta_2+jQ_u\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)} \quad (2)$$

$$S_{21}(\omega) = \frac{2\sqrt{\beta_1\beta_2}}{1+\beta_1+\beta_2+jQ_u\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)} \quad (3)$$

$$S_{22}(\omega) = \frac{1+\beta_1-\beta_2+jQ_u\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)}{1+\beta_1+\beta_2+jQ_u\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)} \quad (4)$$

where the external coupling coefficients β_1 and β_2 at the input and output are defined as

$$\beta_1 = \frac{n_1^2 R_{01}}{R_0}; \quad \beta_2 = \frac{n_2^2 R_{02}}{R_0}. \quad (5)$$

At the resonant angular frequency ω_0, (2)-(4) becomes

$$S_{11}(\omega_0) = \frac{1-\beta_1+\beta_2}{1+\beta_1+\beta_2} \quad (6)$$

$$S_{21}(\omega_0) = \frac{2\sqrt{\beta_1\beta_2}}{1+\beta_1+\beta_2} \quad (7)$$

$$S_{22}(\omega_0) = \frac{1+\beta_1-\beta_2}{1+\beta_1+\beta_2} \quad (8)$$

From (3), (5), and (7), it is straightforward to get

$$Q_L = \frac{\omega_0 L}{R_0 + n_1^2 R_{01} + n_2^2 R_{02}} = \frac{Q_u}{1+\beta_1+\beta_2} = \frac{\omega_0}{\Delta\omega_{3dB}} \quad (9)$$

where $\Delta\omega_{3dB}$ is the 3-dB bandwidth. Eq. (9) is the well-known formula for calculating Q_L from the measured resonant frequency and the 3-dB bandwidth.

Substituting (6) and (8) into (9), we have the expression as follows:

$$Q_u = Q_L(1+\beta_1+\beta_2) = \frac{2Q_L}{S_{11}(\omega_0)+S_{22}(\omega_0)} = \frac{2Q_L}{10^{(-R.L._1/20)}+10^{(-R.L._2/20)}} \quad (10)$$

where $R.L._1(\omega_0)$ and $R.L._2(\omega_0)$ are the return loss at the input and output, respectively, at the resonant frequency. Eq. (10) indicates that by measuring the return losses at the input and output, we can obtain Q_u from the measured Q_L. This measurement method of Q_u is accordingly referred as the return loss method, and the formula (10) as the return loss formula.

If the couplings at the input and output are equal to each other, i.e., if $\beta_1=\beta_2$, from (6) and (8) we have $S_{11}(\omega_0)= S_{22}(\omega_0)$, i.e., $R.L._1(\omega_0)=R.L._2(\omega_0)$. Then by using (6)-(8), formula (10) can be rewritten as

$$Q'_u = \frac{Q_L}{1-S_{21}(\omega_0)} = \frac{Q_L}{1-10^{(-I.L./20)}} \quad (11)$$

Here we use Q'_u to make a difference with Q_u in formula (10). Eq. (11) is the well-know insertion loss formula for measuring the unloaded Q-factor. It reveals that we need only to measure the insertion loss $I.L.(\omega_0)$ of the resonator to determine Q_u from the measured Q_L.

From the above derivation, we see that the insertion formula (11) is valid only when the couplings at the input and output are equal to each other. This assumption is, however, very difficult to be satisfied in many measurements, even careful adjustment of the couplings is made. With unequal couplings at the input and output, the measured Q'_u by the insertion loss formula (11) will have a different value with Q_u measured by the return loss formula (10). The error between Q'_u and Q_u is derived and expressed by the following formula:

$$\frac{Q'_u - Q_u}{Q_u} = \frac{S_{21}(\omega_0) - \sqrt{S_{21}^2(\omega_0) + [S_{11}(\omega_0) - S_{22}(\omega_0)]^2/4}}{1 - S_{21}(\omega_0)} \quad (12)$$

It becomes evident from (12) that the error between Q'_u and Q_u is a function of $S_{21}(\omega_0)$ and the difference $S_{11}(\omega_0)$ - $S_{22}(\omega_0)$. In the case of undercouplings, i.e., when the value of $S_{21}(\omega_0)$ is very small, $Q'_u - Q_u$ becomes approximately proportional to $|S_{11}(\omega_0)-S_{22}(\omega_0)|$.

Numerical results calculated by (12) are drawn in Fig. 2. Instead of $|S_{11}(\omega_0)-S_{22}(\omega_0)|$, the return loss difference $|R.L._1(\omega_0)-R.L._2(\omega_0)|$ in dB is chosen as the x-axis for convenience, because the return loss in dB can be read directly on an network analyzer. In Fig. 2, the insertion loss $I.L.(\omega_0)$ is varied from 5dB to 50dB. It is seen that the error between Q'_u and Q_u increases monotonously with the increase of $|R.L._1(\omega_0)- R.L._2(\omega_0)|$. Also it becomes evident that the larger the insertion loss, the bigger the error between Q'_u and Q_u.

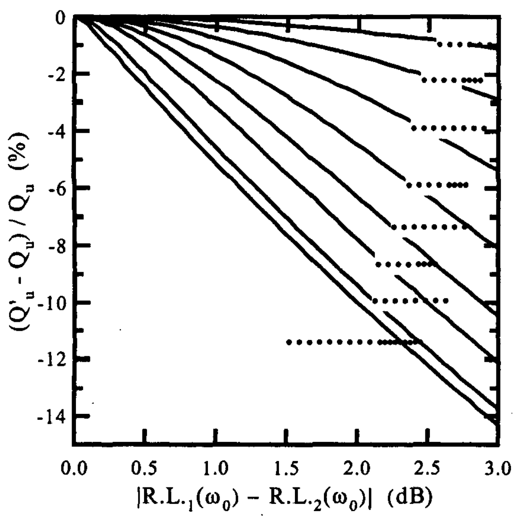

Fig. 2. The calculated error between Q'_u and Q_u as a function of the difference of the return loss, $|R.L._1(\omega_0)-R.L._2(\omega_0)|$. The insertion loss $I.L.(\omega_0)$ is varied from 5dB to 50 dB.

III. MEASUREMENTS

To verify the error formula (12) and the numerical results in Fig. 2, we measured the unloaded Q-factor of a stripline circular patch resonator. The resonator, as shown in Fig. 3, is built by sandwiching a circular conductor patch between two dielectric (TEFLON) substrates. The TEFLON substrates are further sandwiched between two conductor plates. The circular conductor patch has a diameter D=23.5mm and a thickness t=50µm. The TEFLON substrate has a dielectric constant ε_r=2.03 and a thickness d=1.006mm. Coaxial lines coaxially connected to the circular patch are used as the input and output feeds. TM_{010} mode is excited in the resonator for measurements. The couplings at the input and output are controlled by adjusting the distances, L and L', between the coaxial lines and the dielectric substrates as shown in Fig. 3. A photograph of the resonator is given in Fig. 4.

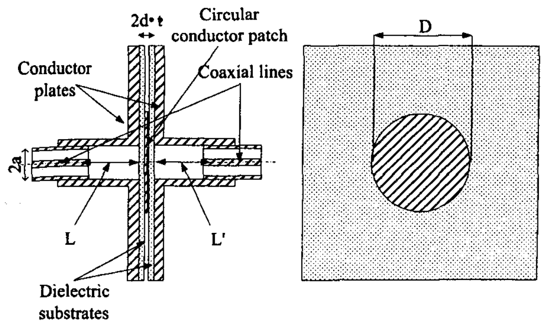

Fig. 3. Configuration of a stripline circular patch resonator operating at TM_{010} mode.

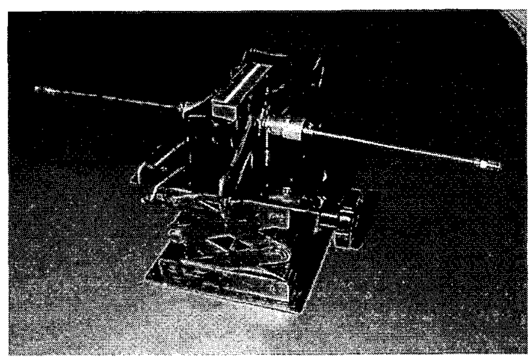

Fig. 4. A photograph of the stripline resonator illustrated in Fig. 3.

The resonator is connected to a network analyzer HP8510B through coaxial cables. The measured resonant frequency is about 10.55 GHz. While keeping the insertion loss $I.L.(\omega_0)$ approximately 20, 30, and 40dB, respectively, we measured three groups of data by varying the return loss at the input and output of the resonator, respectively. From the measured Q_L, the insertion loss $I.L.(\omega_0)$, and the return losses $R.L._1(\omega_0)$ and $R.L._2(\omega_0)$, we get Q_u from (10), Q'_u from (11), and then $Q'_u - Q_u$. The measured errors $(Q'_u - Q_u)/Q_u$ are depicted in Fig. 5 by squares, triangles, and circles, which correspond to $I.L.(\omega_0) \approx 20$, 30, and 40 dB, respectively. The theoretical

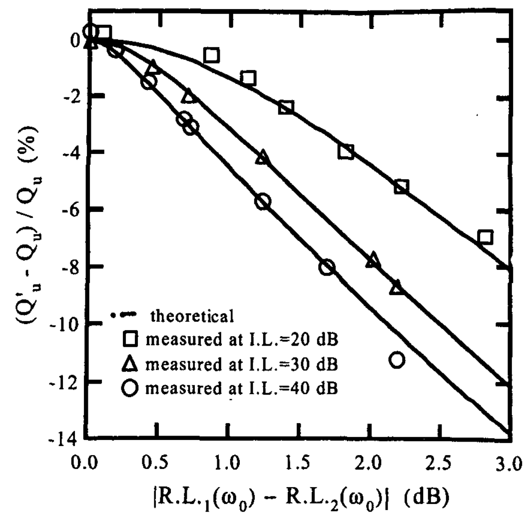

Fig. 5. Comparison of the measured error between Q'_u and Q_u and that predicted by formula (12).

Table 1. Measured Q_L, Q'_u, and Q_u, with varied return loss at the input and output of the stripline resonator. The resonant frequency is about 10.55 GHz, and the insertion loss is kept approximately 20 dB.

I.L. (dB)	f_0 (GHz)	$R.L._1$ (dB)	$R.L._2$ (dB)	Q_L	Q'_u	Q_u
20.21	10.55	0.52	1.38	998.6	998.6	1112.7
20.30	10.55	0.31	2.53	940.1	1040.7	1098.1
20.28	10.55	0.43	1.82	971.7	1075.9	1102.5
20.36	10.55	0.37	2.19	959.0	1060.7	1105.2
20.12	10.55	0.48	1.6	986.7	1094.7	1109.9
20.22	10.55	0.82	0.92	1017.6	1127.5	1124.8
20.16	10.54	0.23	3.05	920.1	1020.3	1096.8
Ave.					1059.8	1107.1
Error					±44.2	±9.7

predictions of $(Q'_u - Q_u)/Q_u$ by (12) are drawn in Fig. 5 by solid lines, and they agree well with the measured data. In Table 1, one group of the measured Q_L, Q'_u, and Q_u are provided. The insertion loss is kept approximately 20 dB. The return losses at the input and output of the stripline resonator are varied for each measurement. Examinations of the values of Q'_u, and Q_u in Table 1 reveal that with unequal couplings at the input and output of the resonator, the value of Q'_u varies in a much larger range (1127.5-998.6=128.9) than that of Q_u (1124.8-1096.8=28.0). The averaged value of Q'_u and Q_u are 1059.8 and 1107.1, respectively, and their errors are • •40.9 and • •9.0, respectively. It is evident that with unequal couplings at the input and output of the resonator, the measured Q_u by the return loss formula has more stable values and smaller errors than the measured Q'_u by the traditionally used insertion loss formula.

IV. Conclusions

The insertion loss method and the return loss method for measuring the unloaded Q-factor of a microwave resonator are reexamined theoretically and compared experimentally. An error formula is derived to estimate the errors between the unloaded Q-factors measured by the two different methods. Measured results of a stripline resonator verified well the derived formula, and proved that the return loss method can provide more accurate and reliable unloaded Q-factors than the traditional insertion loss method.

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